P.G. Semester-II Examination, 2023

PHYSICS

Course ID : 22451 **Course Code : PHS-201C**

Course Title : Mathematical Methods II & Advanced Optics

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

UNIT-I

1. Answer any **three** of the following questions : 2×3=6

   a) Find the Laplace transform of t^{n+1}.

   b) Find the value of $J_{-1}(x)+J_1(x)$.

   c) What is inner product?

   d) Show that $P_n(z)=(-1)^n P_n(-z)$, state its importance.

   e) Show that $L[\sinh at]=\frac{a}{p^z-a^2}$ for p>|a|.

2. Answer any **two** of the following questions : 4×2=8

   a) Solve the initial value problem

   $y''+2y'+y=e^{-t}$, $y(0)=-1$, $y'(0)=1$.

   b) Prove that $H_n(-x)=(-1)^n H_n(x)$.

   c) Show that $\int_{-1}^{1}[P_n(z)]^2 dz=\frac{2}{2n+1}$ Starting from the generating function.

   d) Show that the Fourier transform of a Gaussian function is a Gaussian.

3. Answer any **one** of the following questions : 6×1=6

   a) Consider the following differential equation

   $\frac{d^2y}{dx^2}=R(x)$ with $y(0)=0$ and $y'(1)=0$.

   Find the Green's function required to solve the given differential equation. Solve the equation for $R(x) = \sin x$. 4+2

b) Two function $\psi(x)$ and $\phi(p)$ are Fourier transforms of each other:

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) e^{\frac{ipx}{\hbar}} dp$$

$$\phi(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \psi(x) e^{-\frac{ipx}{\hbar}} dx$$

Show that $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = \int_{-\infty}^{\infty} |\phi(p)|^2 dp$.

The symbols have their usual significance as in Physics.

UNIT-II

4. Answer any **three** of the following questions : 2×3=6

a) What is the difference between MASER and LASER?

b) What are the importance of an optical resonator in a laser?

c) What is the difference between Pockels and Kerr effect?

d) Give two examples of 2nd order nonlinear medium.

e) Determine the MKS units of Einstein's A and B coefficients.

5. Answer any **two** of the following questions: 4×2=8

a) Discuss the working principle of a CO_2 LASER. With a neat energy level diagram indicate the different lasing transitions in CO_2 laser.

b) Discuss the possibility of getting laser from a two level system. How the X-ray laser is obtained?

c) Discuss the working principle and construction of a Bolometer.

d) If two frequency Ω_1 and Ω_2 passes through a second order non-linear medium $x^{(2)}$, then calculate the output frequency components. 4

6. Answer any **one** of the following questions: 6×1=6

a) i) Explain how the population inversion is achieved in He-Ne laser.

ii) Calculate the number of photons emitted from 4.5 mW He-Ne laser.

iii) If the Einstein's A coefficients of the various transitions are $A_{21} = A_{43} = 10^8\ s^{-1}$, $A_{31} = 10^7\ s^{-1}$, and $A_{32} = 7 \times 10^7\ s^{-1}$, calculate the spontaneous lifetime of the metastable state of a 4 level laser. 2+2+2

b) Draw a schematic diagram of three and four energy level LASER system. Discuss how the LASER emission from three level system is possible? 2+4

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